

Varianta 39

III.

13. a) $250 + \frac{10}{100} \cdot 250 = 275$ lei.

b) Prețul final este: $250 + 80 = 330$ lei.

$$275 + \frac{x}{100} \cdot 275 = 330 \Rightarrow \frac{11x}{4} = 55 \Rightarrow x = \frac{4 \cdot 55}{11} \Rightarrow x = 20\%, \text{ deci produsul s-a scumpit cu } 20\%.$$

14. a) $f(0) = -1 \Rightarrow A(0; -1)$; $f(1) = 0 \Rightarrow B(1; 0)$. $g(0) = 3 \Rightarrow C(0; 3)$; $g(1) = 1 \Rightarrow D(1; 1)$.

b) $G_f \cap G_g = \{M\} \Rightarrow f(x) = g(x) \Rightarrow x - 1 = 3 - 2x \Rightarrow x = \frac{4}{3} \Rightarrow f\left(\frac{4}{3}\right) = g\left(\frac{4}{3}\right) = \frac{1}{3} \Rightarrow M\left(\frac{4}{3}; \frac{1}{3}\right)$.

$$A_{COBM} = A_{COE} - A_{BEM} = \frac{CO \cdot OE}{2} - \frac{MF \cdot BE}{2}, \text{ unde } MF \perp Ox \text{ și } E\left(\frac{3}{2}; 0\right). A_{COBM} = \frac{13}{6}.$$

c) $\left. \begin{array}{l} \frac{a-1}{3-2a} \in \mathbf{Z} \Rightarrow \\ \text{dar } 3-2a \mid 3-2a \end{array} \right\} \Rightarrow \left. \begin{array}{l} 3-2a \mid a-1 \\ 3-2a \mid 2a-2 \end{array} \right\} \Rightarrow 3-2a \mid 1 \Rightarrow 3-2a \in \{-1; 1\} \Rightarrow a \in \{1; 2\}.$

15. b) $V_{pir} = \frac{(12\sqrt{3})^2 \cdot 6}{3} = 864 \text{ cm}^3.$

c) $\left. \begin{array}{l} (VBC) \cap (ABC) = BC \\ OM \perp BC, OM \subset (ABC) \\ VM \perp BC, VM \subset (VBC) \\ OM \cap VM = \{M\}, M \in BC \end{array} \right\} \Rightarrow ((VBC), (ABC)) \equiv \widehat{VMO} \Rightarrow m(\widehat{VMO}) = 30^\circ$

d) Dacă $A_{lat.tr.pir.} = 75\% \cdot A_{lat.pir.VABCD} \Rightarrow A_{lat.pir.VA'B'C'D'} = 25\% \cdot A_{lat.pir.VABCD} \Rightarrow$

$$\Rightarrow \frac{A_{lat.pir.VA'B'C'D'}}{A_{lat.pir.VABCD}} = \frac{1}{4} = k^2 \Rightarrow \frac{VO'}{VO} = k = \frac{1}{2} \Rightarrow VO' = OO' = \frac{VO}{2} = 3.$$